① Binomial Distribution

Q → The overall % of failure in a certain examination is 40. What is the probability that out of a group of 6 candidates at least 4 passed the examination.

Solution :- From question

Probability of failure $q = \frac{40}{100}$

$q = \frac{2}{5}$

We know that from the properties of Binomial distribution

probability of success $p = 1 - q$

$= 1 - \frac{2}{5}$

$= \frac{3}{5}$

n = Number of independent trial

= Number of candidates in group

$= 6$

probability of at least 4 passed in the examination $= P(4) + P(5) + P(6)$

$= {}^6C_4 \times \left(\frac{3}{5}\right)^4 \times \left(\frac{2}{5}\right)^{6-4} + {}^6C_5 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^{6-5} + {}^6C_6 \left(\frac{3}{5}\right)^6 \left(\frac{2}{5}\right)^{6-6}$

$= {}^6C_4 \times \left(\frac{3}{5}\right)^4 \times \left(\frac{2}{5}\right)^2 + {}^6C_5 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right) + {}^6C_6 \left(\frac{3}{5}\right)^6 \left(\frac{2}{5}\right)^0$

$= \frac{6!}{4!\,2!} \times \frac{81}{625} \times \frac{4}{25} + \frac{6!}{5!\,1!} \times \frac{243}{3125} \times \frac{2}{5} + 1 \times \frac{729}{15625}$

$= \frac{6 \times 5 \times 4!}{4! \times 2 \times 1} \times \frac{81}{625} \times \frac{4}{25} + \frac{6 \times 5!}{5!} \times \frac{243}{3125} \times \frac{2}{5} + \frac{729}{15625}$

$= \frac{243}{25 \times 125} \times 2 \times 2 + \frac{12 \times 243}{3125 \times 5} + \frac{729}{15625}$

$= \frac{486}{3125} + \frac{243 \times 12}{\ldots}$

$= \frac{4 \times 243}{3125} + \frac{12 \times 243}{15625} + \frac{729}{15625}$

$= \frac{20 \times 243 + 12 \times 243 + 243 \times 3}{15625}$

$= \frac{35 \times 243}{15625}$

$= \frac{8505}{15625}$

$= \frac{1701}{3125}$

(2)

Q → (i) Determine the Binomial distribution for which the mean is 4 and the standard deviation is $\sqrt{3}$.

(ii) Point out the fallacy if any in the following statement "The mean of a Binomial Distribution is 10 and its standard deviation is 4.

Solution :→ From the question

mean = 4

Standard deviation = $\sqrt{3}$,

we know that In the Case of Binomial distribution

where n = total no of trial

p = probability of success

q = probability of failure

$$np = 4$$

$$\sqrt{npq} = \sqrt{3}$$

$$\Rightarrow \frac{\sqrt{npq}}{np} = \frac{\sqrt{3}}{4}$$

$$\Rightarrow \frac{\cancel{np}q}{\cancel{np}} = \frac{3}{4}$$

$$\Rightarrow q = \frac{3}{4}$$

we know that

$$p + q = 1$$

$$p = 1 - \frac{3}{4}$$

$$= \frac{1}{4}$$

$$\Rightarrow np = 4$$

$$n \times \frac{1}{4} = 4$$

$$\Rightarrow n = 16,$$

Binomial distribution

$$p[r] = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{16}C_{r} \left(\frac{1}{4}\right)^{r} \left(\frac{3}{4}\right)^{16-r}$$

where, $r = 0, 1, 2, \ldots 16$

Poisson Distribution (As a limiting case of Binomial distribution)

Poisson distribution may be defined as obtained as a limiting case of Binomial probability distribution under the following condition

[i] n, the number of trial is indefinitely large. i.e $n \to \infty$

(ii) p, the constant probability of success for each trial is very small (indefinitely) i.e $p \to 0$.

(iii) $m = np$ (say) is finite.

The formula of Poisson distribution

$$P(r) = P[X = r] = \frac{e^{-m} m^r}{\lfloor r}$$ where m = mean, r = number of success in trial

$e = 2.71828$

$n \to \infty$, $m = np$, n = number of trial.

$p = \frac{m}{n}$

$q = 1 - \frac{m}{n}$

~~Some Practical~~

There are some practical situation where Poisson distribution can be used.

(a) The number of telephone call arriving at a telephone switch board in unit time (say per minute).

(b) The number of customer arriving at the Super Market (say per hour)

(c) The number of defects / unit of manufactured product

(d) To count the number of bacteria/unit

Mean = m = np
variance
variance = Mean [m]
standard deviation = $\sqrt{m}$
Skewness = $\frac{1}{\sqrt{m}}$, Kurtosis = $\frac{1}{m}$

problem related to Poisson distribution

Q → In a sample of 100 electric bulbs, what are the probabilities of 0, 1, 2, 3, 4 and 5 defective, if it is known from past experience that 2% of the bulbs are found to be defective?

solution → Given that probability of success = 2%
= .02.

we know that

mean m = np
= 100 × .02
= 2

$$P(r) = \frac{e^{-m} m^r}{\lfloor r}$$

$$P(0) = \frac{e^{-2} m^0}{\lfloor 0}$$
$$= e^{-2}$$
$$= 0.13534$$

$$P(1) = \frac{e^{-2} m}{\lfloor 1}$$
$$= 2 \times e^{-2}$$
$$= 2 \times 0.13534$$
$$= 0.27068$$

$$P(2) = \frac{e^{-2} \times m^2}{\lfloor 2}$$
$$= \frac{e^{-2} \times 4}{2}$$
$$= 2 \times e^{-2}$$
$$= 0.27068$$

$$P(3) = \frac{e^{-2} \times m^3}{\lfloor 3}$$
$$= \frac{e^{-2} \times 2^3}{2 \times 3 \times 1}$$
$$= 4 e^{-3}$$

(5)

$$P(4) = \frac{e^{-2} m^4}{\lfloor 4}$$

$$= \frac{e^{-2}\times 2^4}{4\times3\times2\times1}$$

$$= \frac{e^{-2}\times \cancel{16}}{\cancel{4}\times \cancel{6}}$$

$$= \frac{2}{3} e^{-2}$$

$$P(5) = \frac{e^{-2}\times m^5}{\lfloor 5}$$

$$= \frac{e^{-2}\times 2^5}{\cancel{4\times} 5\times3\times2}$$

$$= \frac{e^{-2}\times 2^5}{5\times4\times3\times2\times1}$$

$$= \frac{e^{-2}\times \cancel{32}}{5\times\cancel{4}\times3\times\cancel{2}\times1}$$

$$= \frac{4}{15} e^{-2}$$